Supplement 3—Quantiles (Ch. 3)

Homework 3g. Refer to the data at right.

1. For variable $x$, find:
   a. $Q_1$
   b. $Q_3$
   c. $P_{15}$
   d. $P_{27}$
   e. $P_{67}$
   f. $P_{95}$

2. For variable $y$, find:
   a. median
   b. $P_{25}$
   c. $P_{40}$
   d. $P_{55}$
   e. $P_{79}$
   f. $P_{85}$

Quantile: The value that lies at a specified position in a data array.

Method: Step 1. Find the quantile position:
   
   \[ \text{quantile position} = (n + 1) \times \text{quantile fraction} \]
   
   where $n$ = number of observations
   quantile fraction = .25 for $Q_1$, .67 for $P_{67}$, etc.

   Step 2. If the result in step 1 is not a whole number, find the interpolated value of the quantile:
   
   \[ \text{interpolated value} = \text{lower value} + [\text{upper value} - \text{lower value}] \times \text{excess fraction} \]
   
   where lower value, upper value = values of the variable falling just below and above the quantile position
   excess fraction = .32 if quantile position is 4.32, etc.

Example 1: Find $P_{17}$ for variable $x$ above.

Step 1. \[ P_{17} \text{ position} = (15 + 1) \times .17 = 2.72 \text{th position} \]

*Now, of course there is no 2.72\text{th} value in the data set, so we must interpolate a value between the 2\text{nd} value (lower value = 3) and the 3\text{rd} value (upper value = 5) in step 2.

Step 2. \[ \text{Interpolated value for } P_{17} = 3 + [(5 - 3) \times .72] = 4.44 \]
Hence, $P_{17} = 4.44$

Example 2: Find $P_{25}$ for variable $x$ above.

Step 1. \[ P_{25} \text{ position} = (15 + 1) \times .25 = 4\text{th position} \]

*Since our result in step 1 is a whole number, no interpolation is needed. The value lying at the 4\text{th} position in the array of $x$ is 5.
Hence, $P_{25} = 5$

Example 3: Find $P_{85}$ for variable $x$ above.

Step 1. \[ P_{85} \text{ position} = (15 + 1) \times .85 = 13.6\text{th position} \]

*Our result in step 1 is not a whole number, BUT . . . values 13 and 14 in the array of $x$ are both 12, so there's no need to interpolate.
Hence, $P_{85} = 12$

Example 4: Find $P_{99}$ for variable $x$ above.

Step 1. \[ P_{99} \text{ position} = (15 + 1) \times .99 = 15.84\text{th position} \]

Hey! There are only 15 observations—what gives? Well, this is a quirk of applying our method to small data sets. Just set the $P_{99}$ value equal to the maximum value of $x$, that is, 13.