Supplement 6—addendum to Chapter 6: The constant known as $e$

$e$, or Napier’s Constant (for John Napier, Scottish mathematician), is an irrational number that is the base of the natural logarithmic system. It’s a constant with some truly remarkable characteristics. We’ll not go into all of them here.

Two useful ways to define $e$ are:

$$e = \sum_{i=0}^{+\infty} \frac{1}{i!}$$

That, is $e = 1/0! + 1/1! + 1/2! + 1/3! + \ldots = 2.7182818284590452353602874 \ldots$

Using the concept of the limit, $e$ can be defined as

$$\lim_{m \to +\infty} e = \left(1 + \frac{1}{m}\right)^m$$

The idea here is that we start with a single “thing,” represented by the 1. Then we add to it the smallest imaginable quantity, which is $1/m$ as our closest approach to $1/+\infty$. We then multiply this by itself, well, over and over, endlessly. (Try it on your calculator: Plug in 100, then 1,000,000, then 1,000,000,000, for m, and notice that the results will approach 2.7182818284590452353602874 \ldots

This explains why $e$ lies at the heart of endogenous growth models. (Endogenous means generated internally, as opposed to exogenous, generated externally.) The growth of virus populations can be modeled with this, since a virus population starts with one virus, which then multiplies itself into new viruses, which then multiply themselves, which then \ldots Present value and compound interest formulas use it, too. If you deposit a sum of money into a bank account paying continuously compounded interest, then each millisecond, a miniscule amount of interest is earned, which then is added to the existing principle, upon which a miniscule (but ever so slightly larger) amount of interest is earned, and so on. Very useful, indeed.

Why does the Poisson distribution, which is a discrete distribution, involve $e$, which is a calculus-based, and hence continuous-distribution, concept? Because the origins of the Poisson distribution lie in calculus, specifically, the law of rare events. For a nice discussion, see Wikipedia (http://en.wikipedia.org) under the topic “Poisson Distribution.”

Finding the value of functions of $e$ on the calculator

1. Backwards-entry (cheap) calculators:
   - $e^{3.5}$ : $[\text{2nd}] \Rightarrow [\text{ln}] \Rightarrow [-/+] \Rightarrow 3 \Rightarrow [\text{5}] \Rightarrow [e^x]$
   and 0.03019738 displays

2. Algebraic-entry calculators (typical):
   - $e^{3.5}$ : $[\text{2nd}] \Rightarrow [\text{ln}] \Rightarrow [-/+] \Rightarrow 3 \Rightarrow [\text{5}] \Rightarrow [e^x]$
   and 0.03019738 displays